# Semaine 12
# Doutes

ME-332 – Mécanique Vibratoire

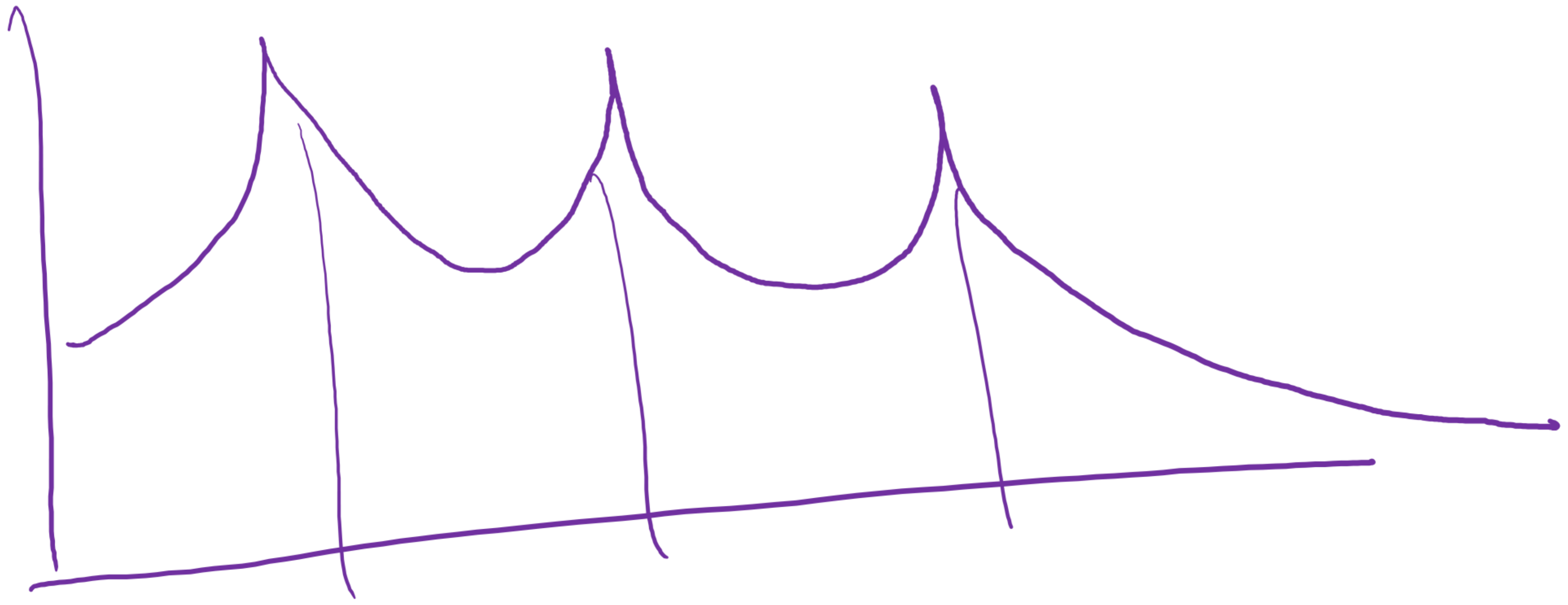
École
polytechnique
fédérale
de Lausanne

# Doutes

$\vec{X} = (Y) \cdot \vec{F}$

$\vec{X} = D \cdot \vec{q}$

$Y$



Sem. 3 $\rightarrow$ $F = F_0 \cdot \cos(\omega t)$

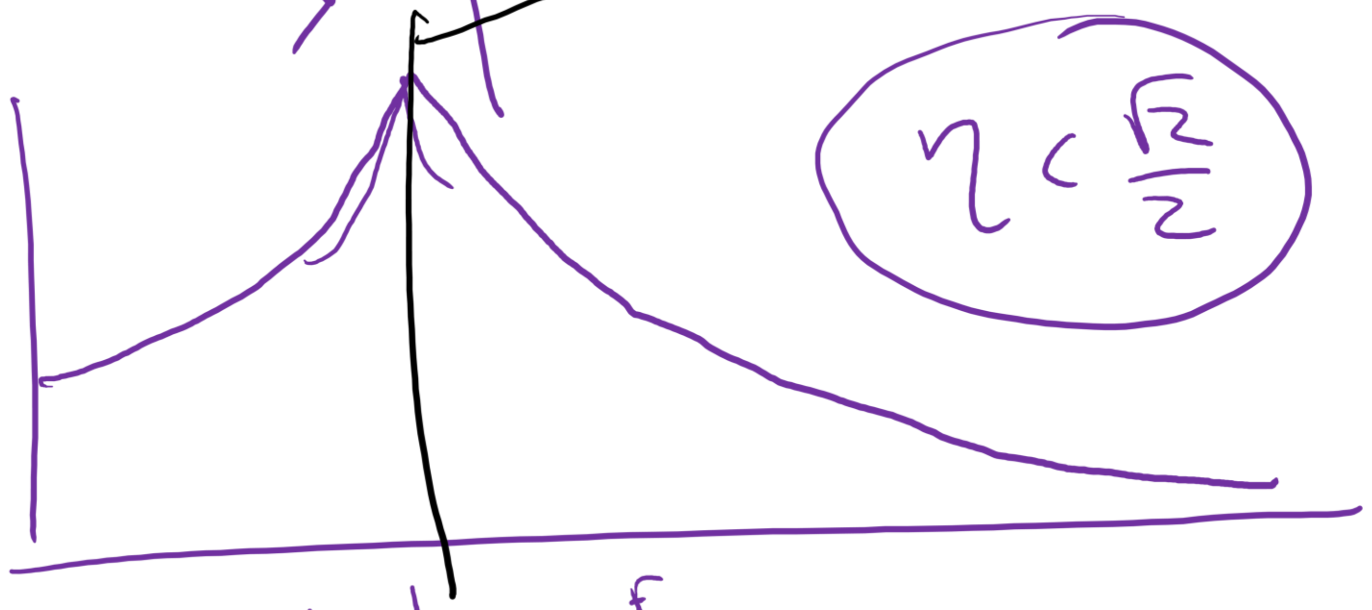
$x(t) = \frac{\mu(\omega)}{k} \cdot F_0 \cdot \cos(\omega t - \varphi)$

$\underset{\sim}{x}(t) = \left(\frac{\mu(\omega)}{k} e^{-j\varphi}\right) \underbrace{F_0 \, e^{j\omega t}}_{\underset{\sim}{F}(t)}$

$Y$

$\omega_2 = \omega_0 \sqrt{1 - 2\eta^2}$

$\eta < \frac{\sqrt{2}}{2}$



$m\ddot{x} + c\dot{x} + kx = f$

$\underset{\sim}{x} = \underset{\sim}{F} \cdot \frac{1}{\left(\frac{k}{m} - \omega^2 + j\frac{c\omega}{m}\right) m} = \frac{F}{m(\omega_0^2 - \omega^2 + j2\eta\omega_0\omega)}$

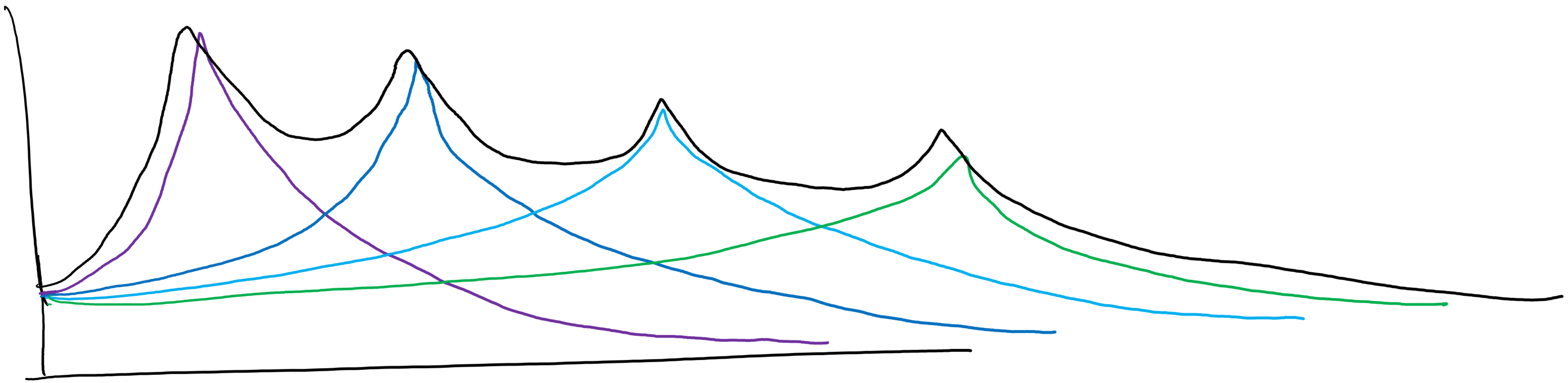
Mécanique Vibratoire – ME332 - SGM Ba5 - G. Villanueva

# Doutes

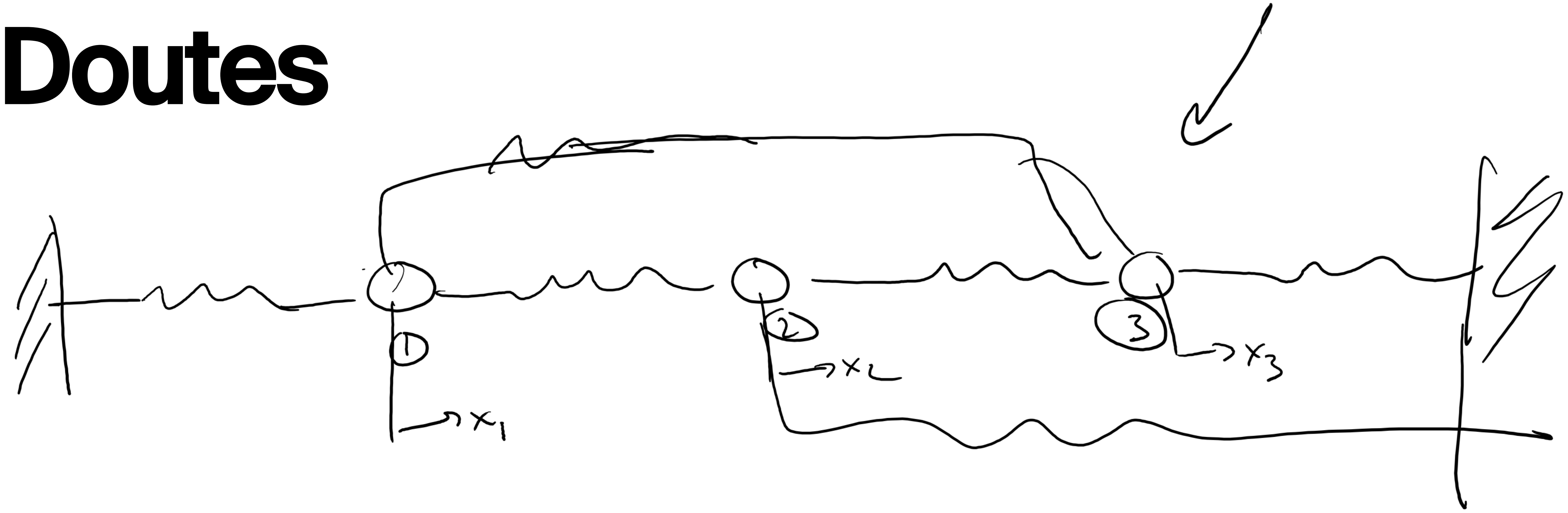
$$[Y](j\omega) = \frac{\vec{X}(j\omega)}{\vec{F}(j\omega)} = \sum_{p=1}^{n} \frac{\vec{\beta}_p \otimes \vec{\beta}_p}{m_p^0(-\omega^2 + 2j\eta_p\omega_{0p}\omega + {\omega_{0p}}^2)}$$

$$Y_{rs}(j\omega) = \frac{X_r(j\omega)}{F_s(j\omega)} = \sum_{p=1}^{n} \frac{\beta_r^p \beta_s^p}{m_p^0(-\omega^2 + 2j\eta_p\omega_{0p}\omega + {\omega_{0p}}^2)}$$

I II III IV



# Doutes




$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \vec{\beta}_I = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$\vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad \vec{\beta}_{II} = \begin{pmatrix} \pi \\ 0 \\ -\pi \end{pmatrix}$$

$$\vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \qquad \vec{\beta}_{III} = \begin{pmatrix} 1.5 \\ -3 \\ 1.5 \end{pmatrix}$$

$$\vec{F}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot F_0$$

$$\vec{F}^0 = B^T \cdot \vec{F}$$

Force sur 1er mode :

$$\vec{F}^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} F_0 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \Rightarrow$$

$$B = \begin{pmatrix} \vec{\beta}_I & \vec{\beta}_{II} & \vec{\beta}_{III} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{pmatrix} ; \quad B^T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{pmatrix}$$

$$F_1 - F_3 = 0 \Rightarrow F_1 = F_3$$

$$F_1 - 2F_2 + F_1 = 0 \; ; \; F_1 = F_2$$

$$F_1 + F_2 + F_3 = 3F_1 = F_0$$

$$F_1 = F_0/3$$

$$\vec{F}^0 = \begin{pmatrix} F_0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \vec{F} = \begin{pmatrix} F_0/3 \\ F_0/3 \\ F_0/3 \end{pmatrix}$$

# Semaine 12
# QCMs

ME-332 – Mécanique Vibratoire

École
polytechnique
fédérale
de Lausanne

# Plan du cours

| Week | Jours | Partie du cours | Exercices | Homework |
|---|---|---|---|---|
| 1 | 09.09//11.09 | Intro et oscillateur 1D libre conservatif | Série 1 | Leçon 1.1&1.2 |
| 2 | 18.09 | Oscillateur 1D libre dissipatif | Série 2 | Leçon 2.1&2.2 |
| 3 | 23.09//25.09 | Régime permanent harmonique (complexe) | Série 3 | Leçon 3.1&3.2 |
| 4 | 30.09//02.10 | Régime permanent périodique (série de Fourier) | Série 4 | Leçon 4.1&4.2 |
| 5 | 07.10//09.10 | Régime forcé général (transf. Laplace ou Fourier) | Série 5 | Leçon 5.1&5.2 |
| 6 | 14.10//16.10 | Systèmes à 2 DDL conservatifs | Série 6 | Leçon 6.1&6.2 |
| BREAK!!! | | | | |
| 7 | 28.10//30.10 | Récapitulatif // séance de QCMs | **MIDTERM** | |
| 8 | 04.11//06.11 | Systèmes à 2DDL dissipatifs, osc. de Frahm | Série 7 | Leçon 7.1&7.2 |
| 9 | 11.11//13.11 | Systèmes à n DDL, conservatif | Série 8a | Leçon 8.1&8.2 |
| 10 | 18.11//20.11 | Systèmes à n DDL, conservatif | Série 8b | |
| 11 | 25.11//27.11 | Systèmes à n DDL, dissipatif | Série 9 | Leçon 9.1&9.2 |
| 12 | 02.12//04.12 | Systèmes à n DDL, forcé | Série 10 | Leçon 10.1&10.2 |
| 13 | 09.12//11.12 | Systèmes continus: barres & poutres | Série 11 | Leçon 11.1&11.2 |
| 14 | 16.12//18.12 | Récapitulatif // séances de QCMs | | |

# Before we start… let's connect!



- Smartphones
  - iPhone
  - Android

  Download and install the app "*PointSolutions*"

  pointsolutions
  powered by echo360

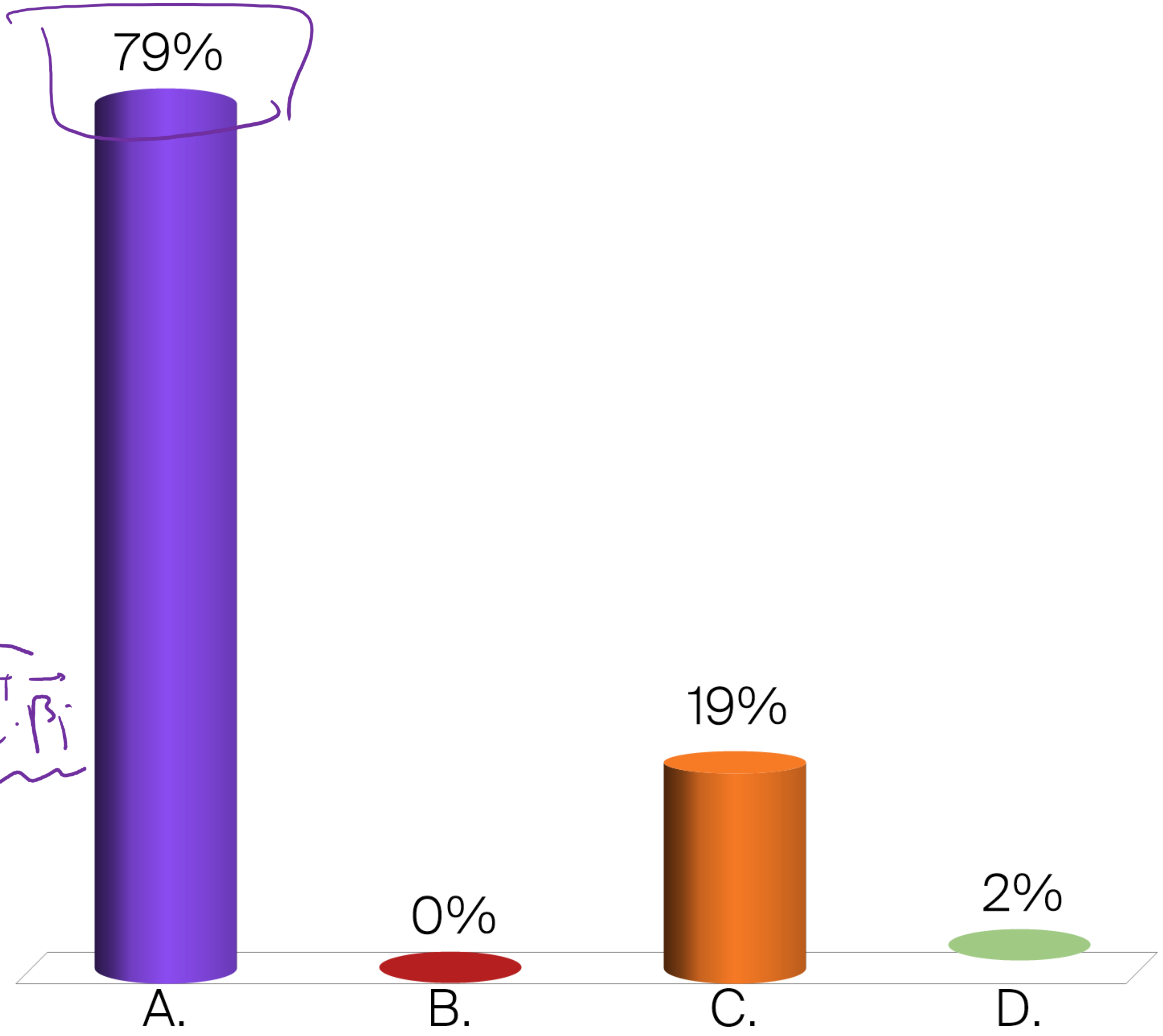
- Computers (or Windows Phones)
  - Go to [www.responseware.eu](http://www.responseware.eu)

- Choose **EUROPE Server** (if asked)
- Join session – **ME332**
- When asked – **do not input your name. Enter as anonymous.**

# Système avec $M = m\mathbb{I}$, $\vec{\beta}$ ortogonaux?

$B = \left(\vec{\beta}_I, \vec{\beta}_{II}, \ldots \vec{\beta}_N\right)$

A. Oui
B. Non
C. Ça dépend
D. Je ne sais pas




$M \rightarrow$ réelles $\rightarrow \{x_1, x_2, x_3 \ldots\}$

$M^\circ \rightarrow$ nodales ; $M^\circ = \begin{pmatrix} m_I & & & 0 \\ & m_{II} & & \\ & & \ddots & \\ 0 & & & m_N \end{pmatrix}$

$M^\circ = B^t M B$

$M^\circ_{ij} = B_i^T \cdot M \cdot B_j = \vec{\beta}_i^{\,t} \cdot M \cdot \vec{\beta}_j = m \cdot \vec{\beta}_i^{\,T} \cdot \vec{\beta}_j$

$i \neq j \Rightarrow M^\circ_{ij} = 0 = m \cdot \vec{\beta}_i^{\,T} \cdot \vec{\beta}_j$

$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ ; $\vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. **Combien DdL?**

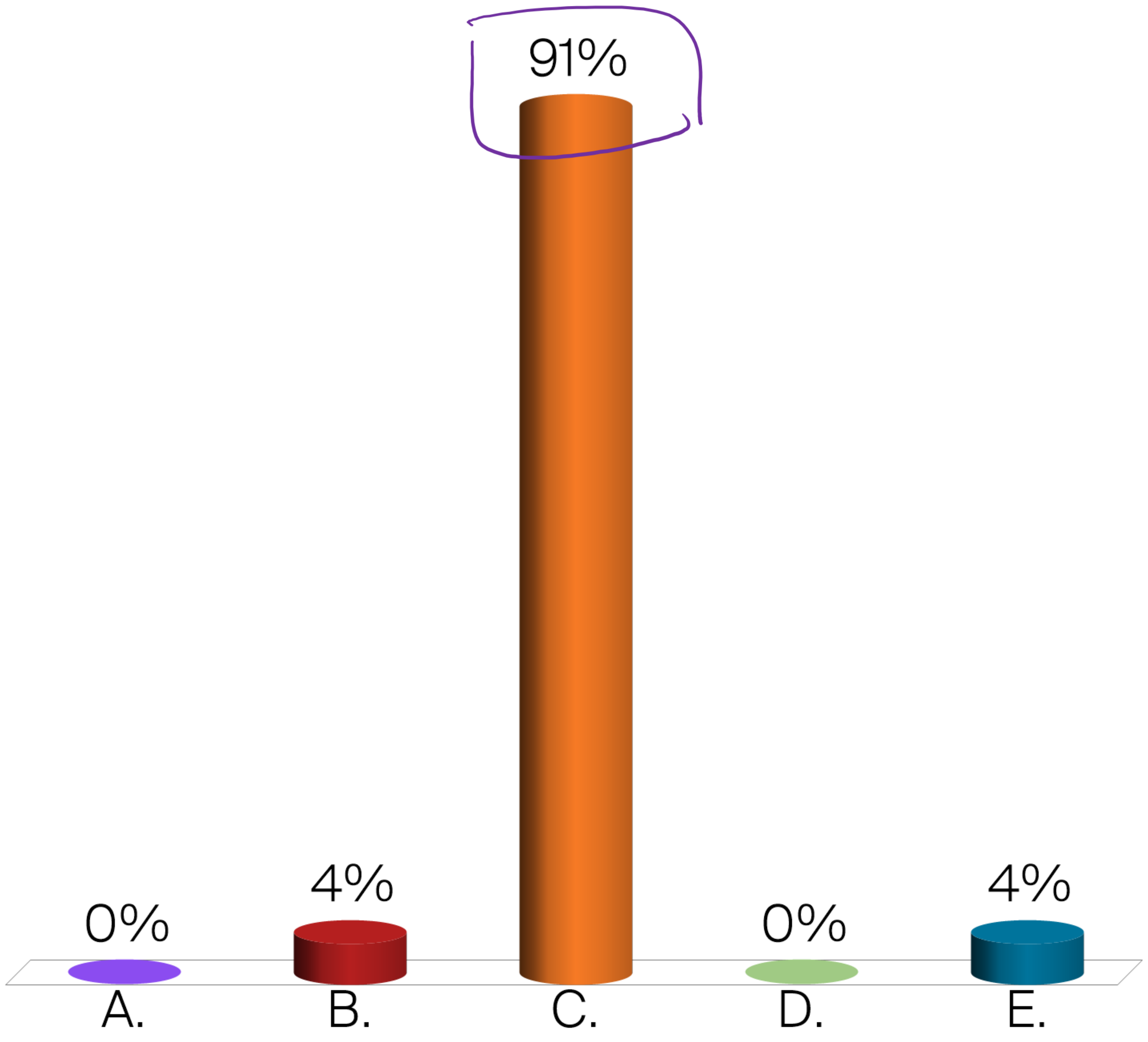
A. 1

B. 2

C. 3

D. 4

E. Je ne sais pas




Mécanique Vibratoire - ME332 - SGM Ba5 - G. Villanueva

EPFL

# $\vec{\beta}_{II}$?

$M = m\,\bar{I}$

$\vec{w} = \vec{u} \times \vec{v}$

$$\vec{\beta}_I = \begin{pmatrix}1\\1\\1\end{pmatrix} \; ; \; \vec{\beta}_{III} = \begin{pmatrix}1\\-2\\1\end{pmatrix}$$

A. $\vec{\beta}_{II} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$

B. $\vec{\beta}_{II} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$

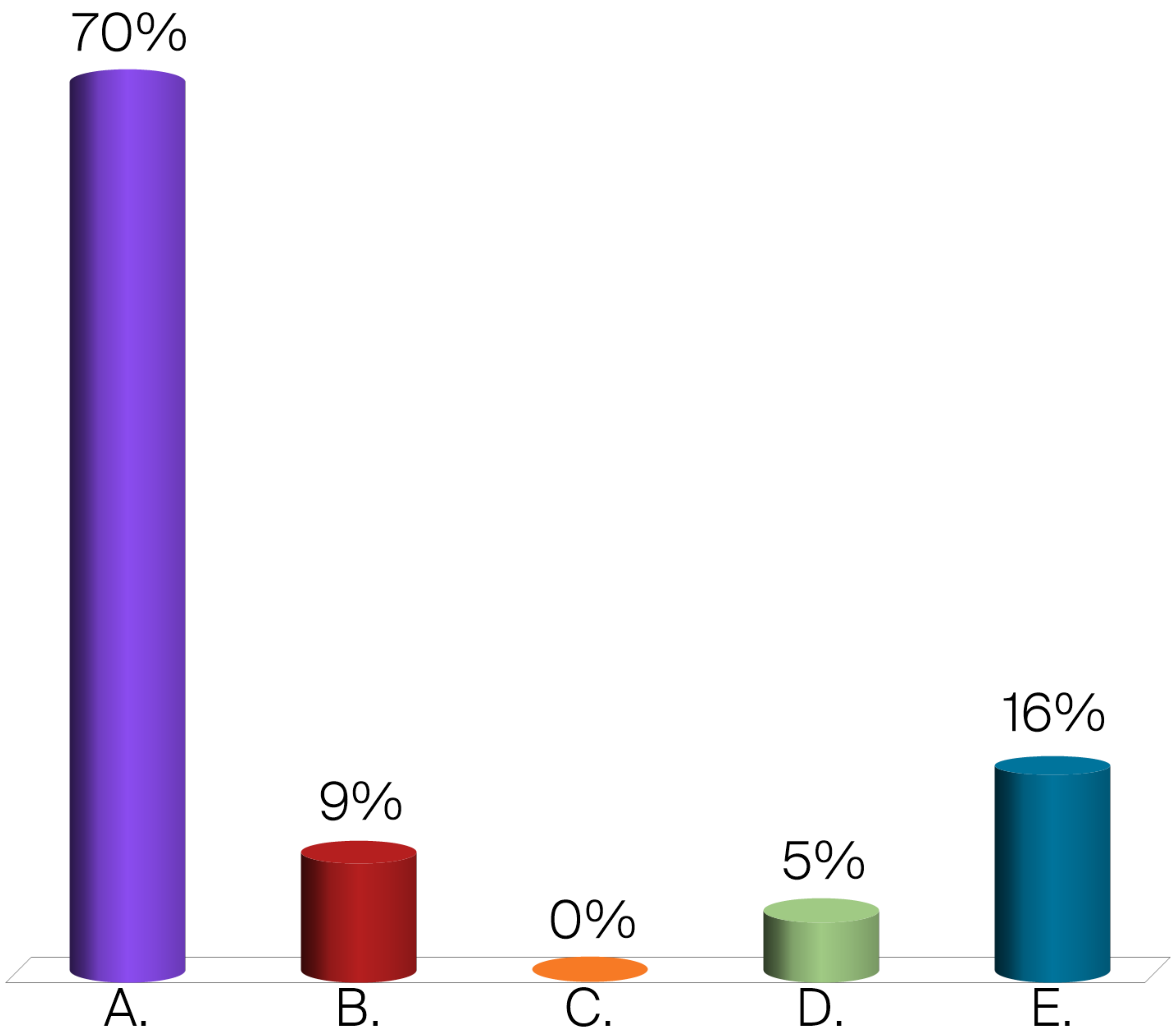
C. $\vec{\beta}_{II} = \begin{pmatrix}1\\0\\1\end{pmatrix}$

D. $\vec{\beta}_{II} = \begin{pmatrix}2\\0\\-2\end{pmatrix}$

E. Je ne sais pas

$\begin{pmatrix}\pi\\0\\-\pi\end{pmatrix}$

$\begin{pmatrix}100000\\0\\-100000\end{pmatrix}$



# Matrice de masses en coordonnées modales?

$\underline{M = m\mathbb{I}}$ $\quad \vec{\Phi}_I = \begin{pmatrix}1\\1\\1\end{pmatrix} \quad \vec{\Phi}_{II} = \begin{pmatrix}1\\-2\\1\end{pmatrix}$

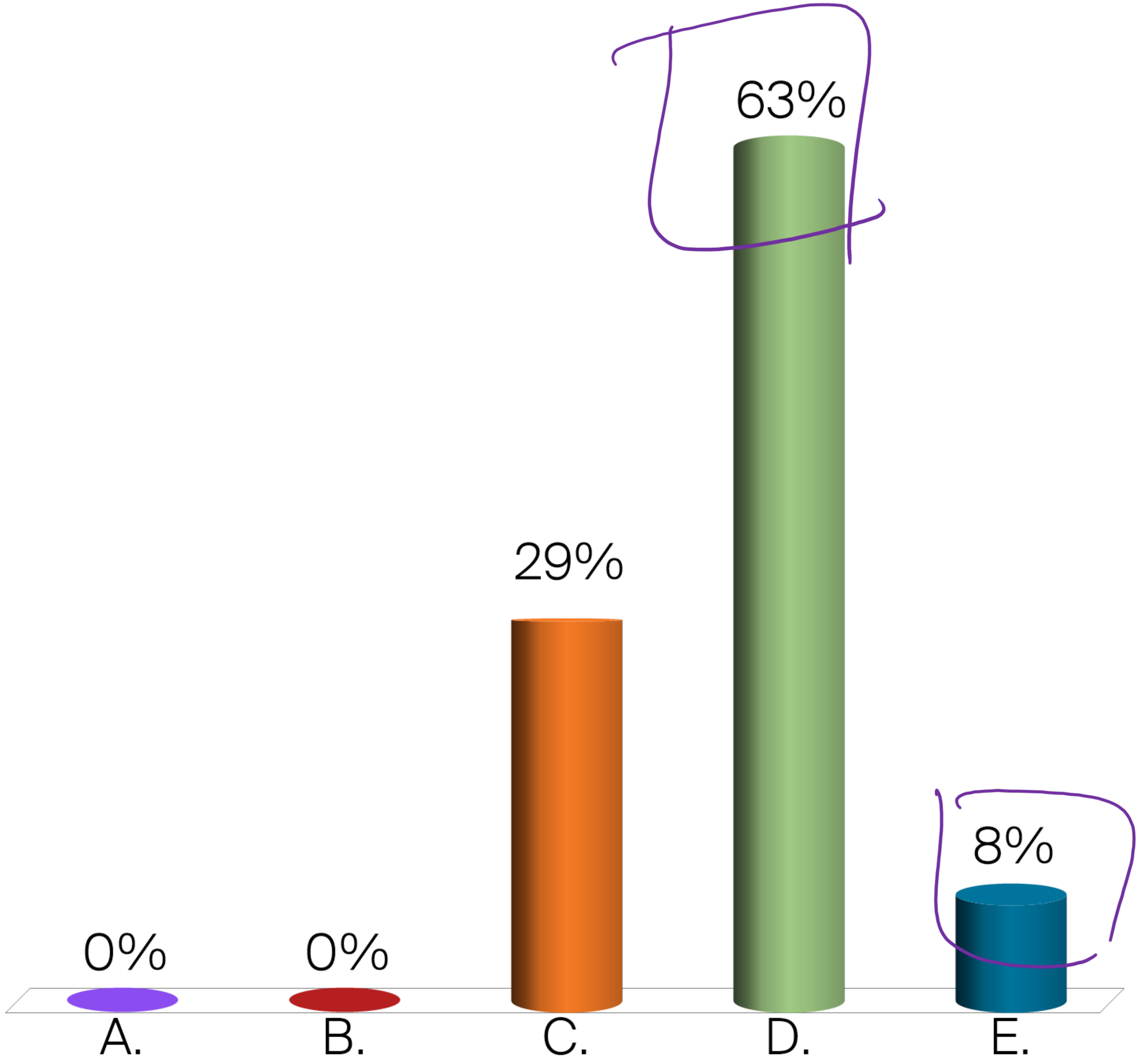
A. $M^0 = m\mathbb{I}$

B. $M^0 = 2m\mathbb{I}$

C. $M^0 = m\begin{pmatrix}3 & 0 & 0\\0 & 2 & 0\\0 & 0 & 6\end{pmatrix}$

D. Ça dépend

E. Je ne sais pas



# EPFL Matrice de masses en coordonnées modales?

$M = m\mathbb{I}$

$$\vec{\beta}_I = \begin{pmatrix}1\\1\\1\end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix}1\\0\\-1\end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix}1\\-2\\1\end{pmatrix} \rightarrow B = \begin{pmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{pmatrix}$$

A. $M^0 = m\mathbb{I}$

B. $M^0 = 2m\mathbb{I}$

C. $\boxed{M^0 = m\begin{pmatrix}3 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 6\end{pmatrix}}$

D. Ça dépend

E. Je ne sais pas

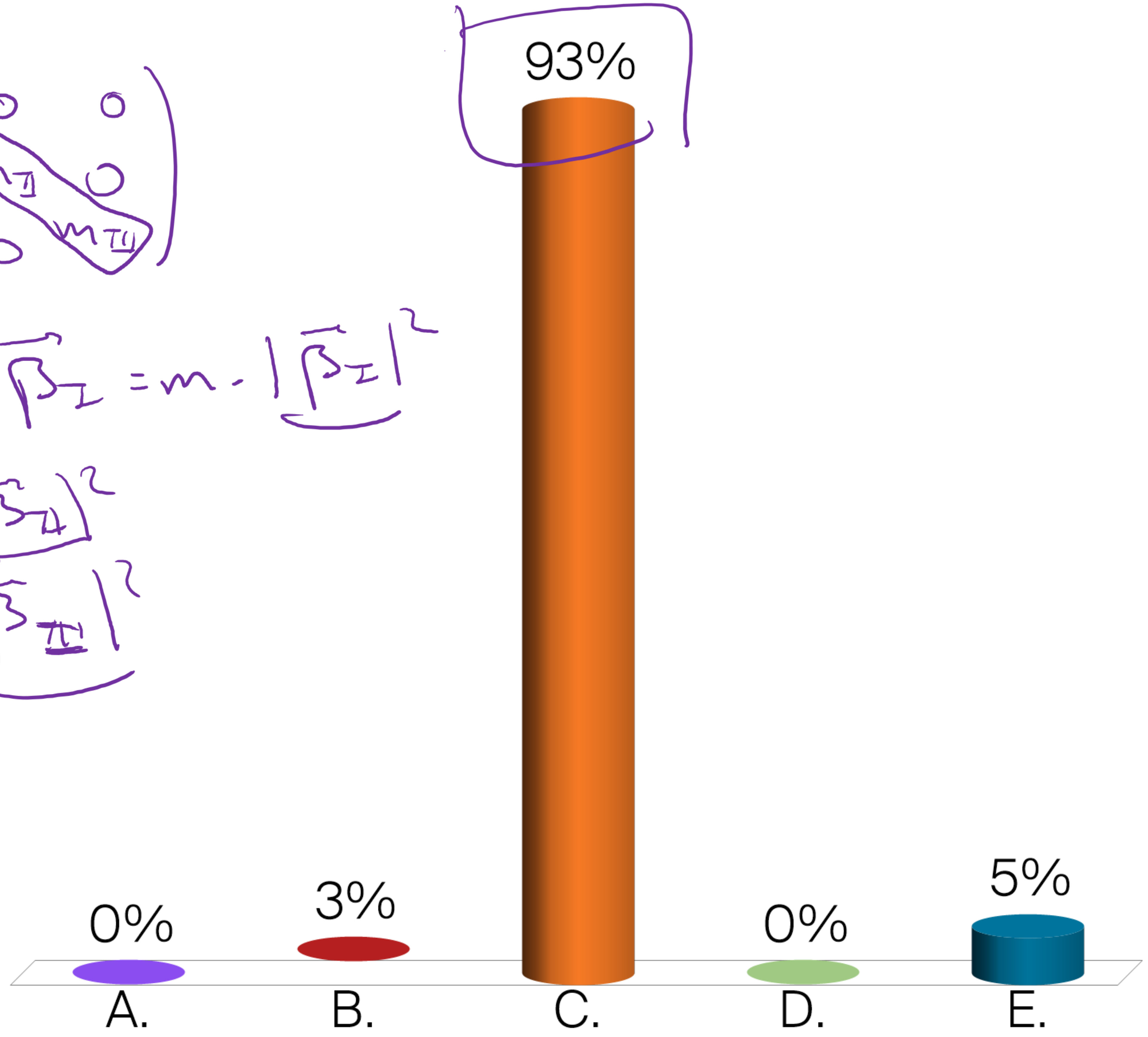
$M^0 = B^T M B$

$$M^0 = \begin{pmatrix}m_I & 0 & 0\\ 0 & m_{II} & 0\\ 0 & 0 & m_{III}\end{pmatrix}$$

$m_I = \vec{\beta}_I^T \cdot M \cdot \vec{\beta}_I = m \cdot |\vec{\beta}_I|^2$

$m_{II} = m \cdot |\vec{\beta}_{II}|^2$

$m_{III} = m \cdot |\vec{\beta}_{III}|^2$



# Matrice de rigidité en coordonnées modales?



A. $K^0 = k\mathbb{I}$

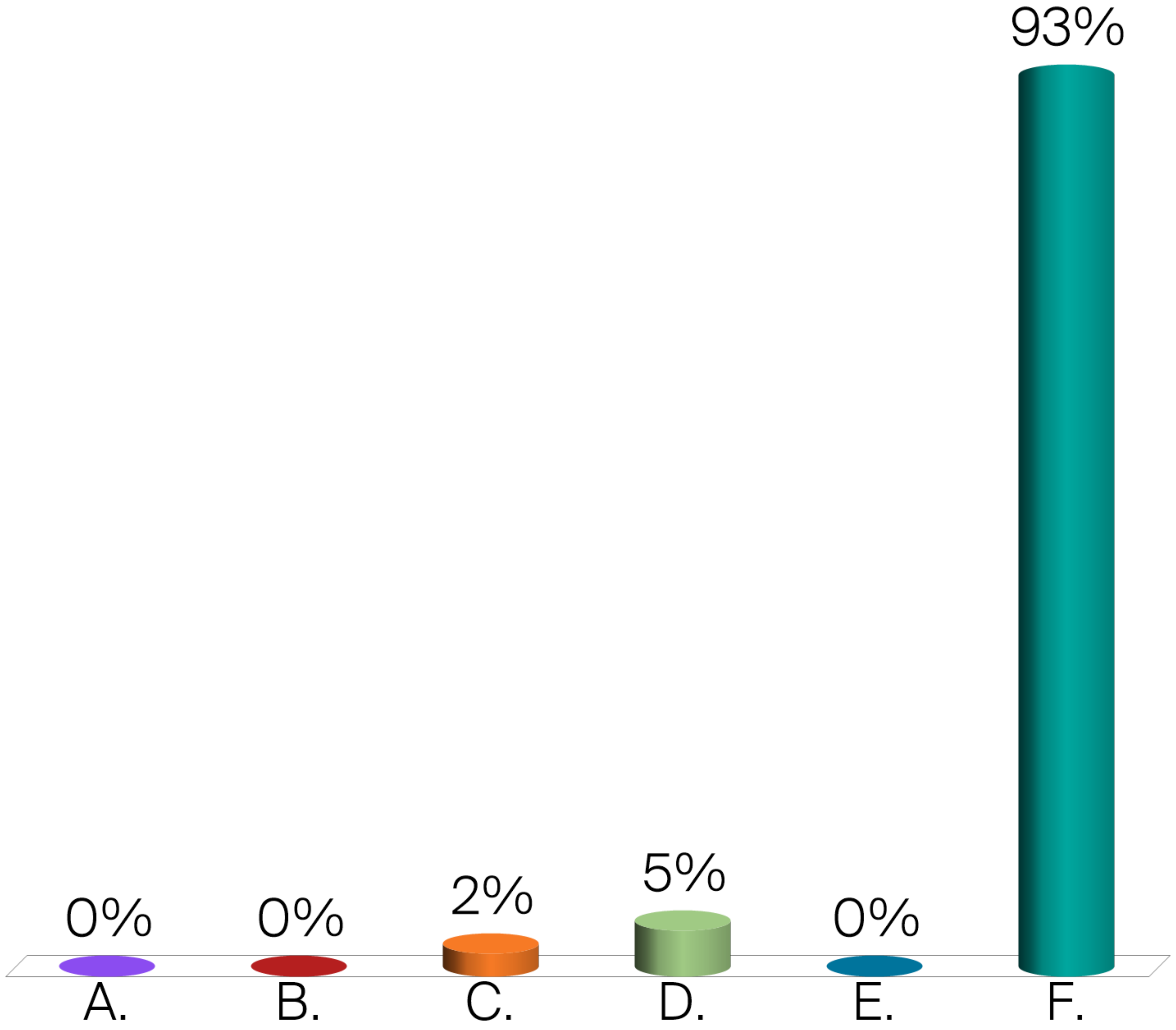
B. $K^0 = 2k\mathbb{I}$

C. $K^0 = k\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

D. Ça dépend

E. Je ne sais pas

F. On a besoin de plus d'info




 www.responseware.eu

# Matrice de rigidité en coordonnées modales?

$$\omega_{I,0} = \sqrt{\frac{k}{m}}, \omega_{II,0} = \sqrt{\frac{2k}{m}}, \omega_{III,0} = 2\sqrt{\frac{k}{m}}$$

$M^0, K^0$ ??

A. $K^0 = k\mathbb{I}$

B. $K^0 = 2k\mathbb{I}$

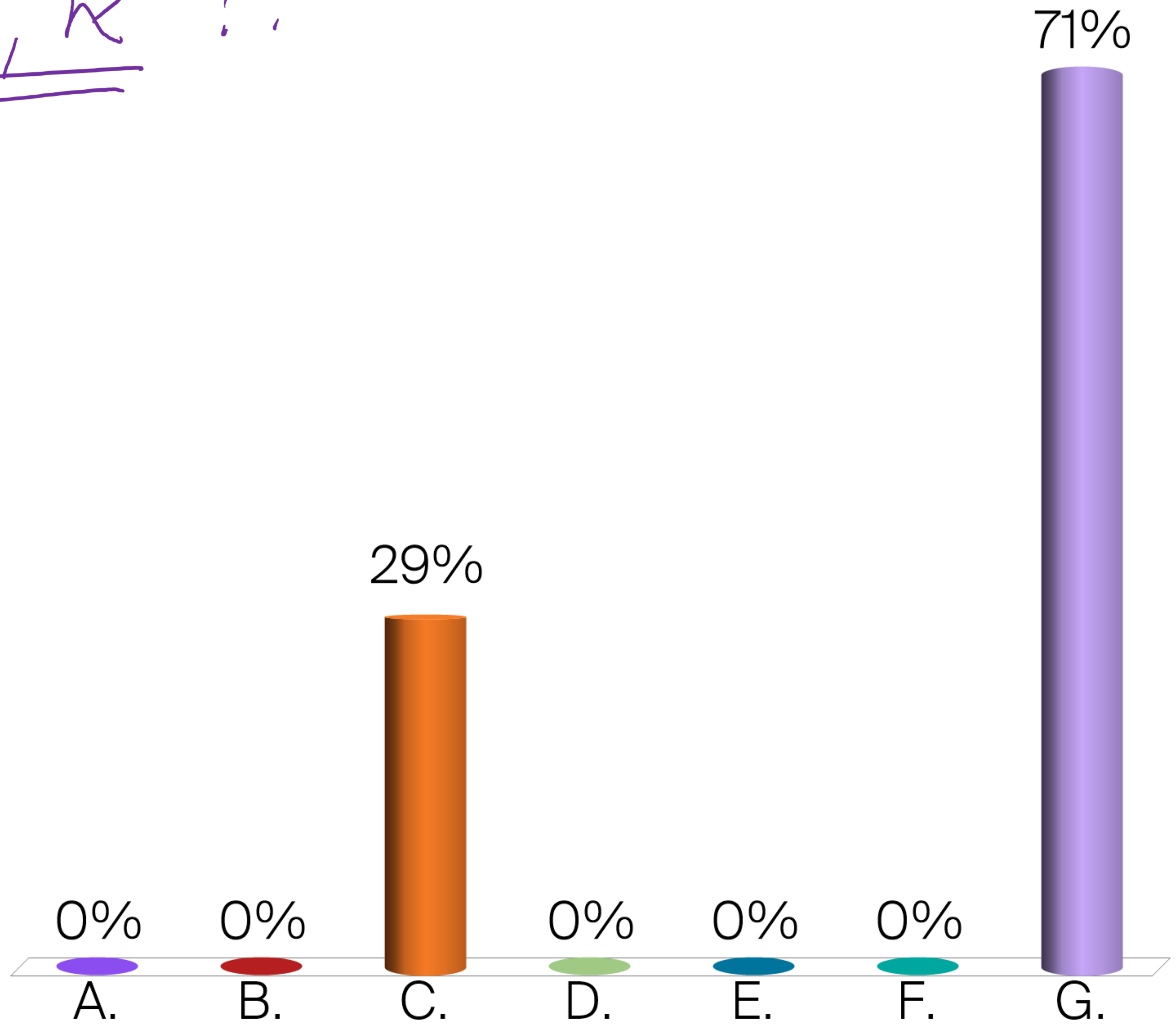
C. $K^0 = k\begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 24 \end{pmatrix}$

D. $K^0 = k\begin{pmatrix} 9 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

E. Ça dépend

F. Je ne sais pas

G. Pour quelles vecteurs propres?



# Matrice de rigidité en coordonnées modales?

$$\omega_{I,0} = \sqrt{\frac{k}{m}}, \omega_{II,0} = \sqrt{\frac{2k}{m}}, \omega_{III,0} = 2\sqrt{\frac{k}{m}}$$

$$\vec{\beta}_I = \begin{pmatrix}1\\1\\1\end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix}1\\0\\-1\end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix}1\\-2\\1\end{pmatrix}$$

$$M^0 = m\begin{pmatrix}3&0&0\\0&2&0\\0&0&6\end{pmatrix}$$

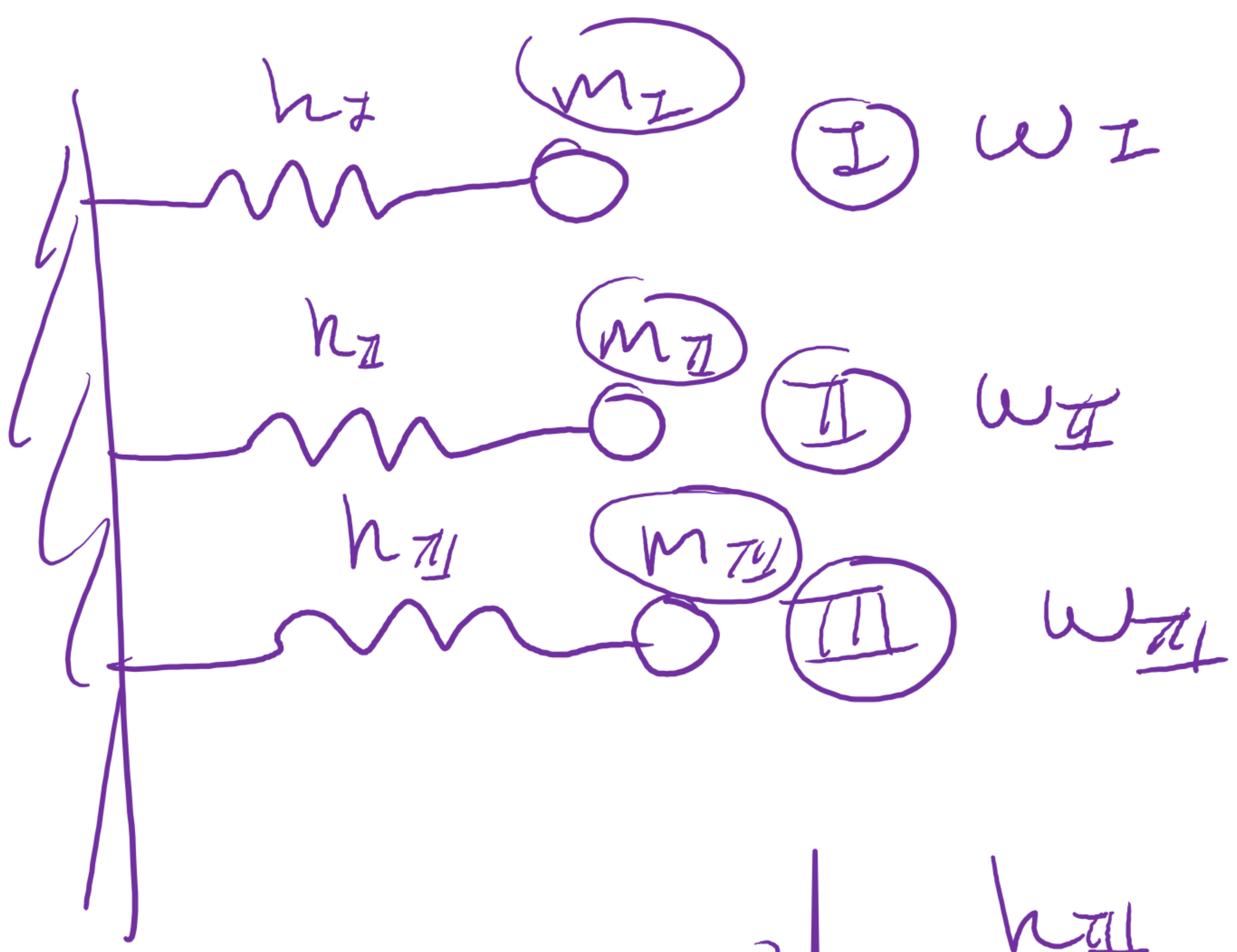
A. $K^0 = k\mathbb{I}$

B. $K^0 = 2k\mathbb{I}$

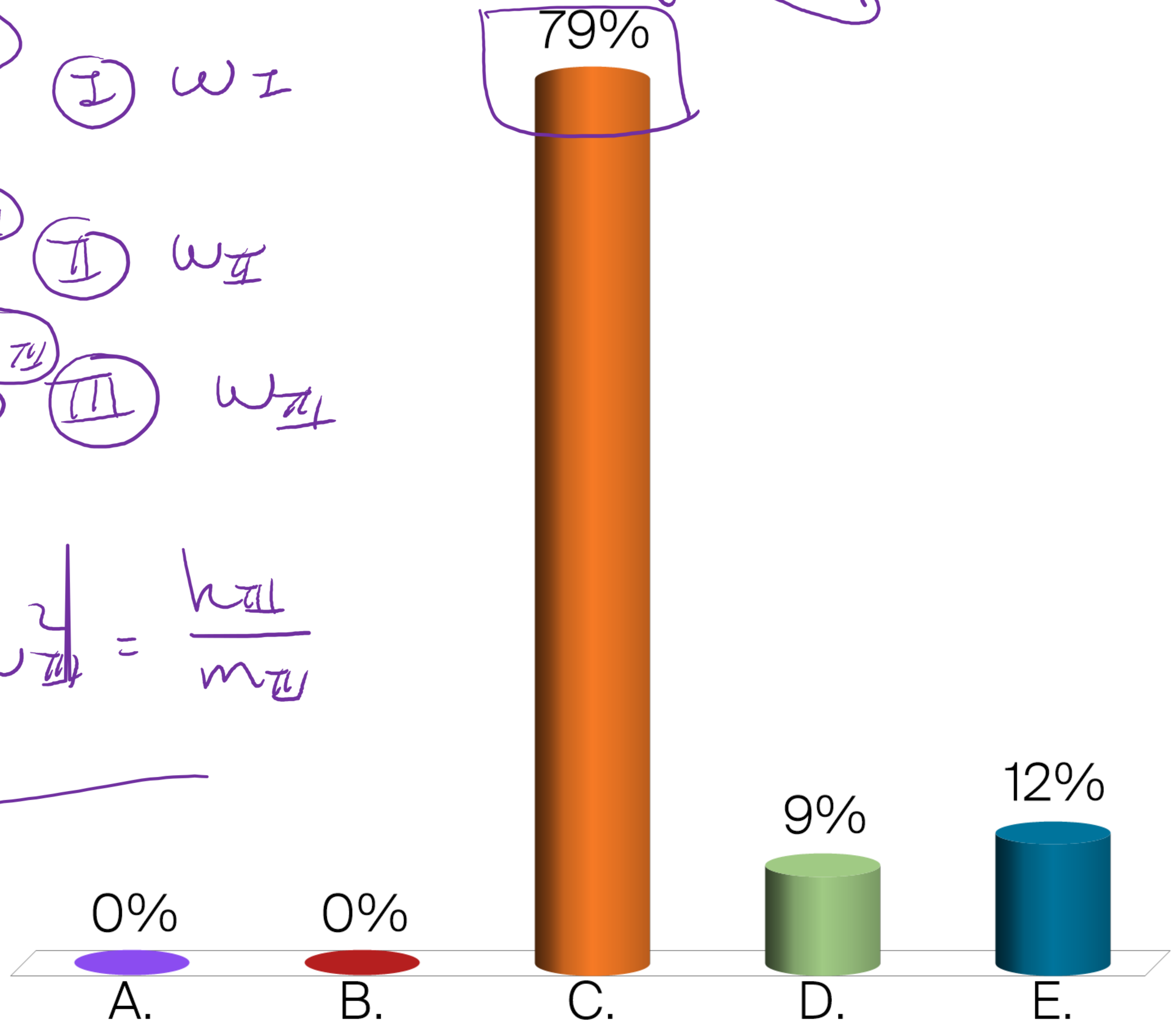
C. $K^0 = k\begin{pmatrix}3&0&0\\0&4&0\\0&0&24\end{pmatrix}$

D. $K^0 = k\begin{pmatrix}9&0&0\\0&12&0\\0&0&6\end{pmatrix}$

E. Je ne sais pas




$$\omega_I^2 = \frac{k_I}{m_I};\quad \omega_{II}^2 = \frac{k_{II}}{m_{II}};\quad \omega_{III}^2 = \frac{k_{III}}{m_{III}}$$



# Matrice de rigidité en coordonnées «réelles»?

$\boxed{M = m\mathbb{I}}$ $\qquad$ $\boxed{M^0 = m\begin{pmatrix}3 & & 0\\ 0 & 2 & \\ & & 6\end{pmatrix}}$ $\qquad$ $\boxed{K^0 = k\begin{pmatrix}3 & & 0\\ 0 & 4 & \\ & & 24\end{pmatrix}}$

A. $K = k\mathbb{I}$

B. $K = 2k\mathbb{I}$

C. $K = B^{-1}K^0B$

D. $\boxed{K = B^{T^{-1}}K^0B^{-1}}$

E. $K = B^{-1}K^0B^{T^{-1}}$

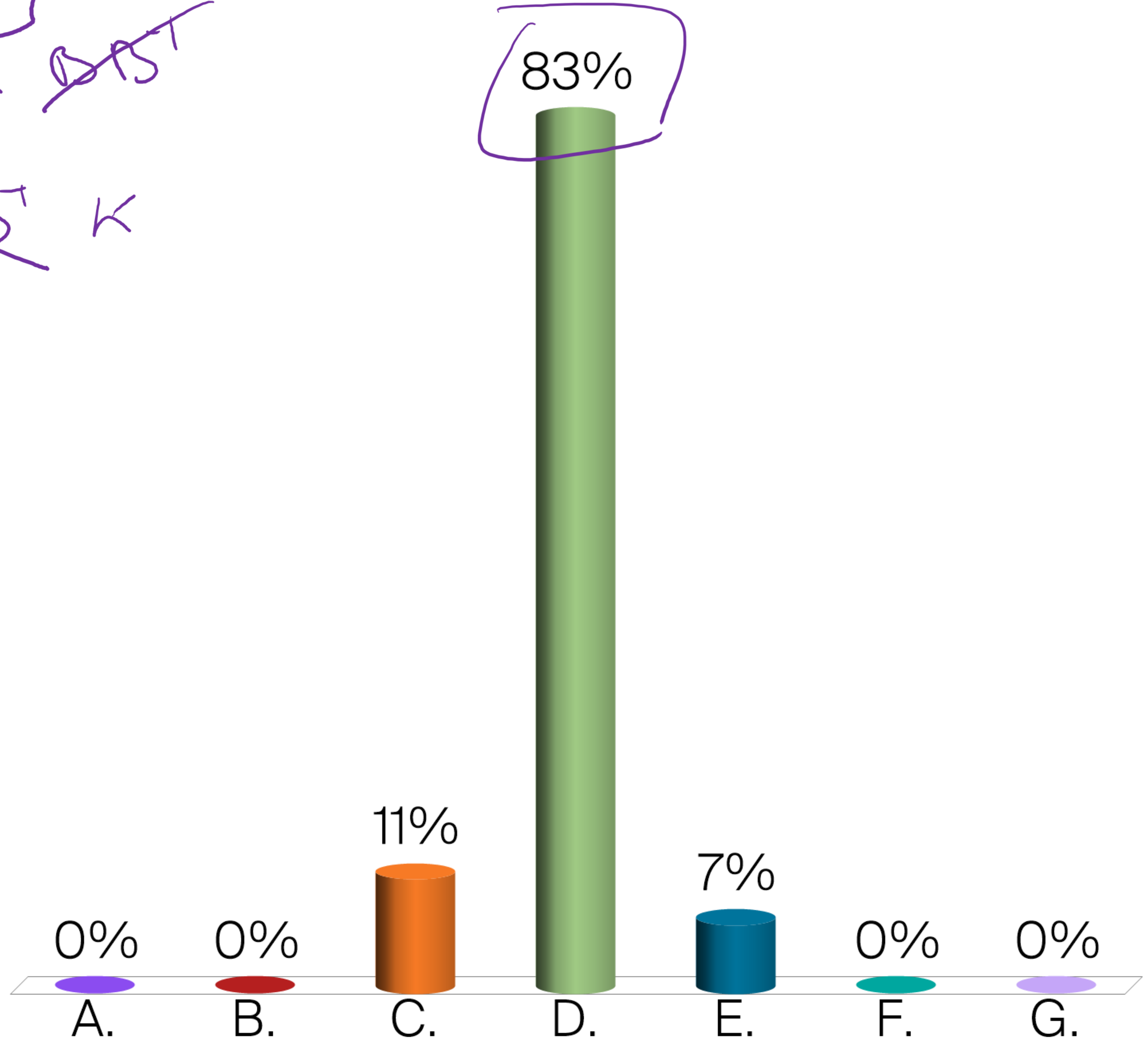
F. Ça dépend

G. Je ne sais pas

$\boxed{K^0 = B^T K \cdot B}$

$K^0 B^{-1} = B^T K \cancel{B B^{-1}}$

$B^{T^{-1}} K^0 B^{-1} = \cancel{B^{T^{-1}} B^T} K$



# EPFL Matrice noyau dans coordonnées «réelles»?

A. $A = B^{-1}\Delta B$

B. $A = B\Delta B^{-1}$

C. $A = B^{T^{-1}}\Delta B^{-1}$

D. $A = B^{-1}\Delta B^{T^{-1}}$

E. Ça dépend

F. Je ne sais pas

$A = M^{-1}K$

$\underline{\Delta} = \underline{M^{\circ -1}K^{\circ}} = B^{-1}\cdot M^{-1}\cdot \underbrace{B^{T-1}\cdot B}_{I}\cdot K\cdot B = B^{-1}\underbrace{(M^{-1}\cdot K)}_{A}\cdot B$

$\underline{\Delta} = B^{-1}M^{-1}KB = \underline{B^{-1}AB}$

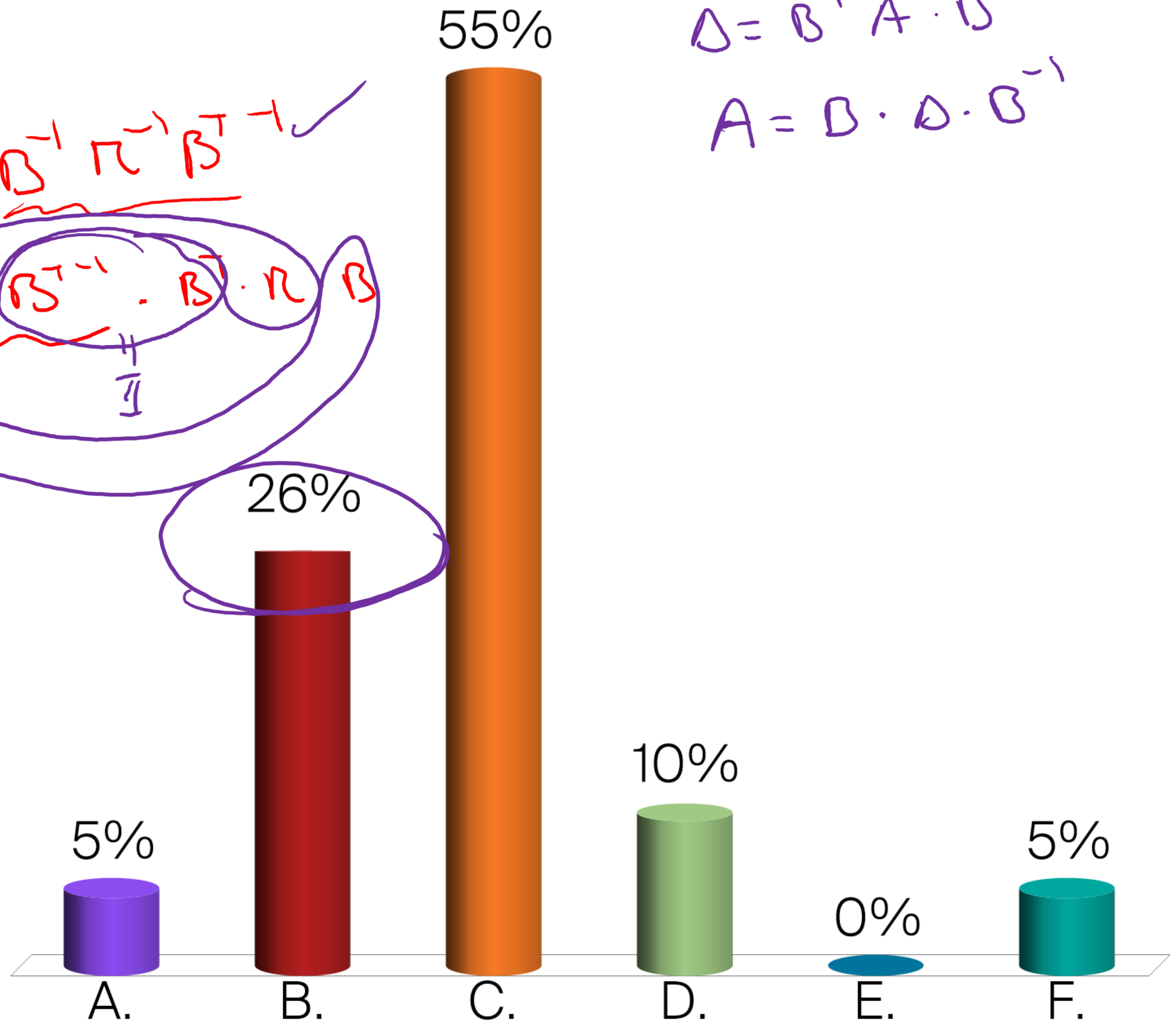
$K^{\circ} = B^{T}KB$

$M^{\circ} = B^{T}MB \Rightarrow M^{\circ -1} = B^{-1}M^{-1}B^{T-1}$ ✓

$M^{\circ -1}\cdot M^{\circ} = I = B^{-1}M^{-1}\underbrace{B^{T-1}\cdot B^{T}}_{I}\cdot MB$

$\Delta = B^{-1}A\cdot B$

$A = B\cdot\Delta\cdot B^{-1}$



# Matrice noyau dans coordonnées modales?

$$\omega_{I,0} = \sqrt{\frac{k}{m}}, \omega_{II,0} = \sqrt{\frac{2k}{m}}, \omega_{III,0} = 2\sqrt{\frac{k}{m}}$$

A. $\cancel{\Delta = \frac{k}{m}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}}$

B. $\Delta = \frac{k}{m}\mathbb{I}$

C. $\Delta = \frac{k}{m}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

D. $\boxed{\Delta = \frac{k}{m}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}}$

E. Ça dépend

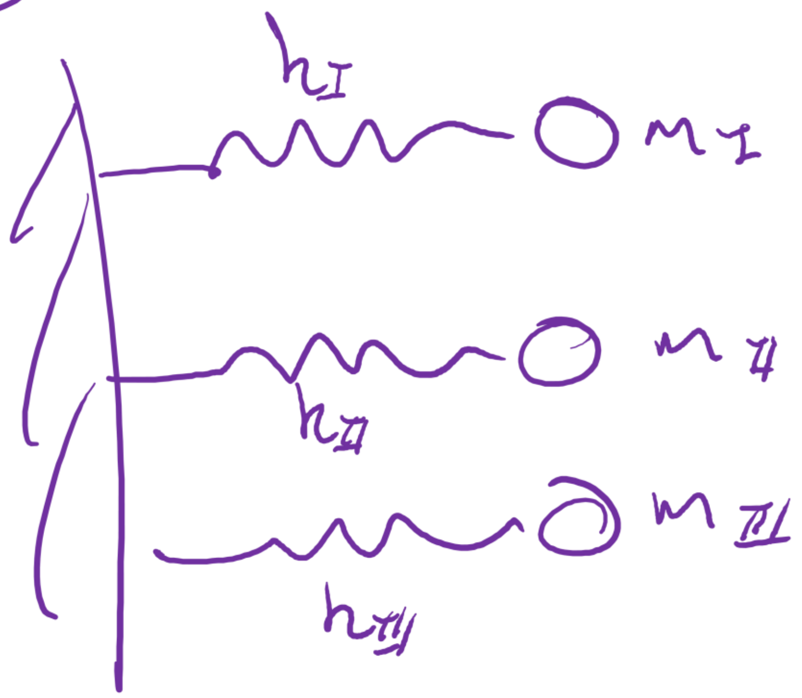
F. Je ne sais pas

$\Delta = M^{0-1} \cdot K^0$

$\Delta = D^{-1} A D$

- $M^0 \rightarrow$ Diag. $\rightarrow$ Pas unique
- $K^0 \rightarrow$ Diag. $\rightarrow$ Pas unique

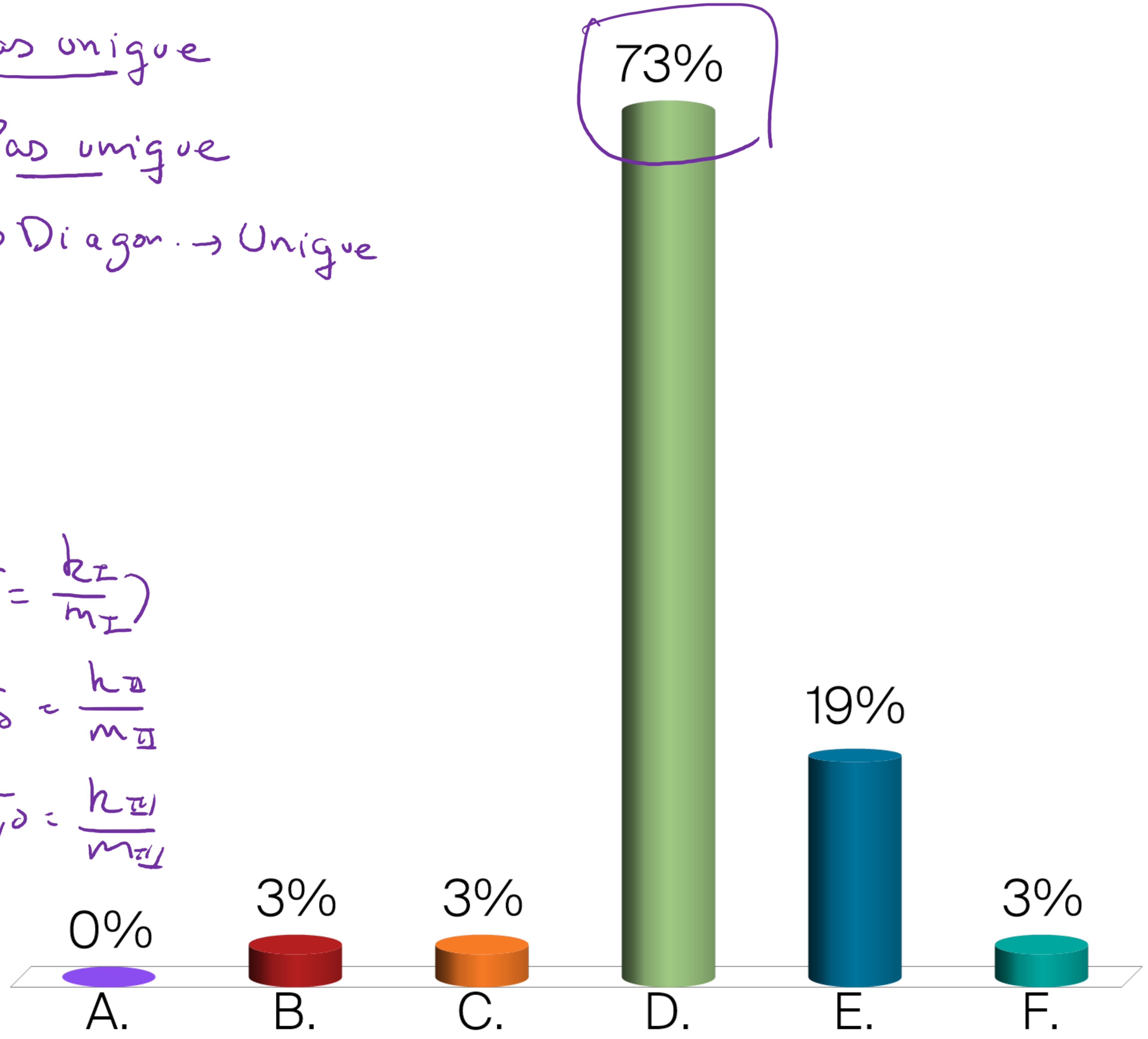
$\Delta = M^{0-1} K^0 \rightarrow$ Diagon. $\rightarrow$ Unique




$\omega_{I,0}^2 = \frac{k_I}{m_I}$

$\omega_{II,0}^2 = \frac{k_{II}}{m_{II}}$

$\omega_{III,0}^2 = \frac{k_{III}}{m_{III}}$



# Masses effectives?

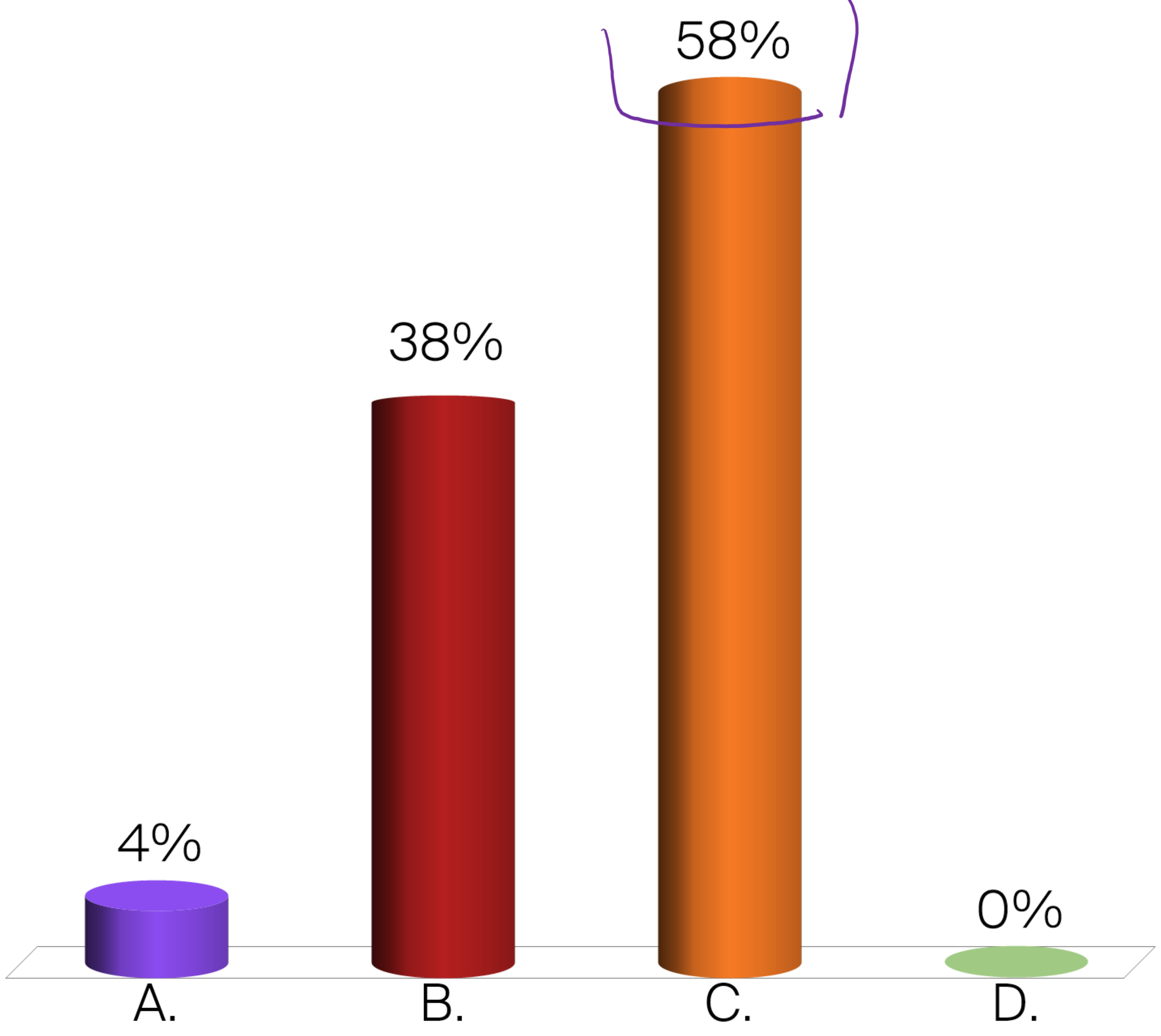
$m_I = m_{II}$

A. $m_I = 3m; m_{II} = 2m; m_{III} = 6m$

B. $m_I = m; m_{II} = m; m_{III} = m$

C. Ça dépend

D. Je ne sais pas



# Masses effectives?

$M = m\,I$

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \ \vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}; \ \vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$M^0 = B^T \cdot M B = m \cdot B^T \cdot B$$

A. $m_I = 3m; m_{II} = 2m; m_{III} = 6m$

B. $\cancel{m_I = 3; m_{II} = 2; m_{III} = 6}$

C. $\cancel{m_I = 1; m_{II} = 1; m_{III} = 1}$

D. $m_I = m; m_{II} = m; m_{III} = m$

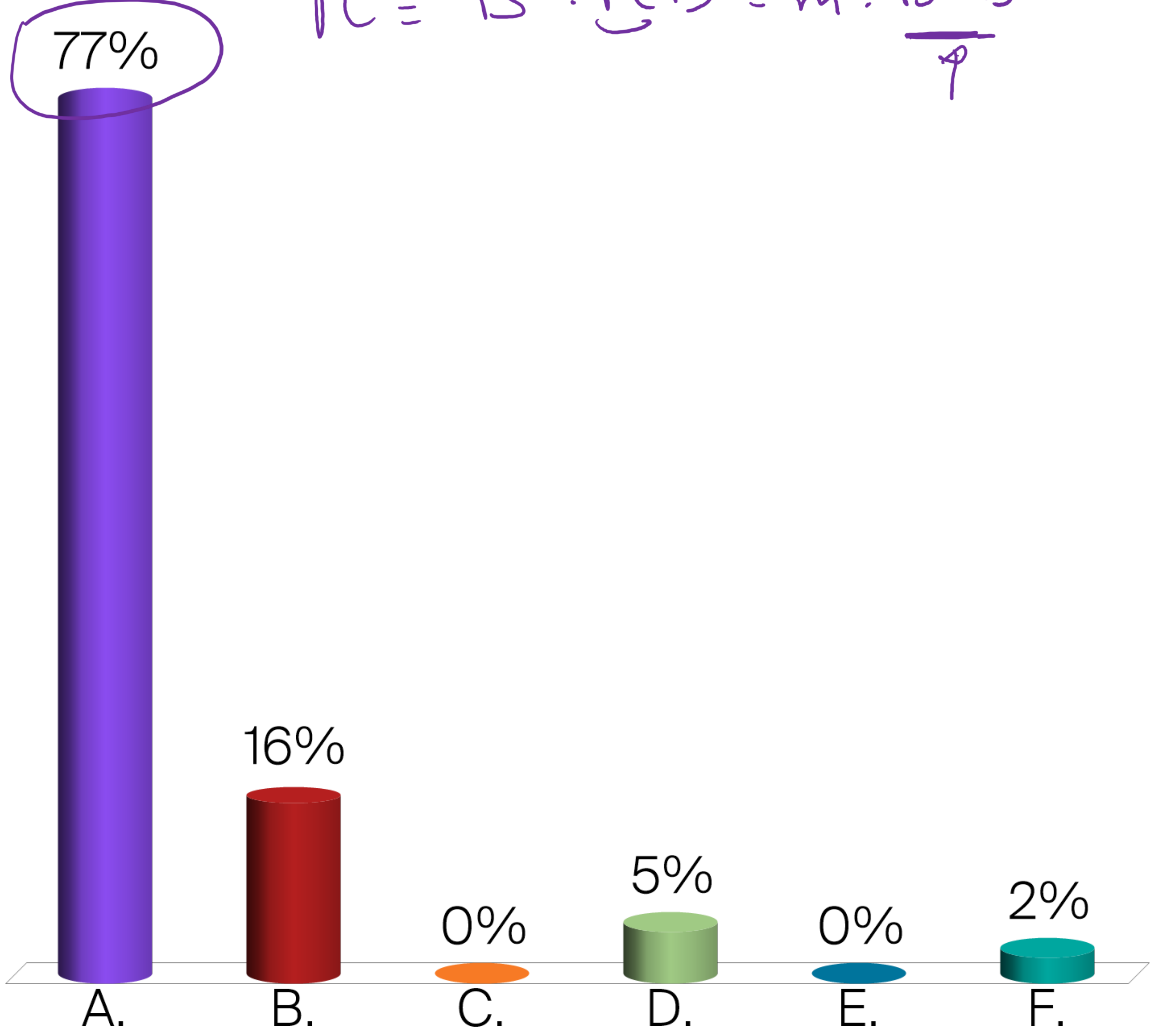
~~E. Ça dépend~~

F. Je ne sais pas

$m_I = \vec{\beta}_I^T \cdot M \cdot \vec{\beta}_I = m \cdot |\vec{\beta}_I|^2 = 3m$

$m_{II} = \vec{\beta}_{II}^T \cdot M \cdot \vec{\beta}_{II}$

$m_{III} = \vec{\beta}_{III}^T \cdot M \cdot \vec{\beta}_{III}$

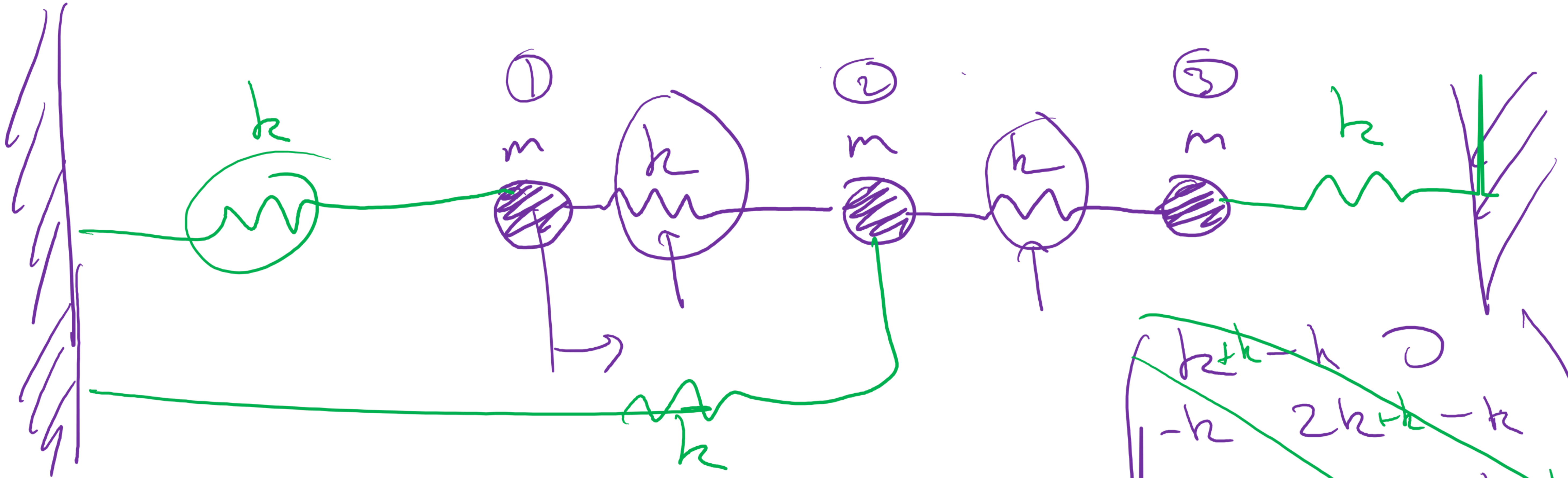


# Propose un schéma pour un système qui correspond au problème

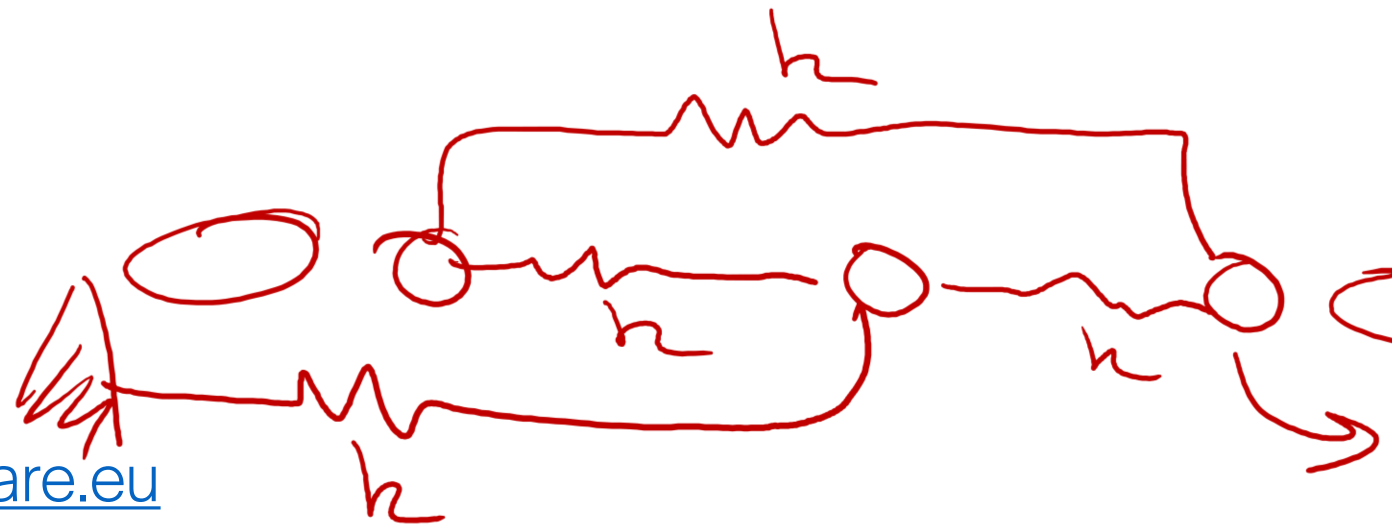
$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$K = k\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$M = m\mathbb{I}$$




$$\begin{pmatrix} k+k & -k & 0 \\ -k & 2k+k & -k \\ 0 & -k & k+k \end{pmatrix}$$




$$k\begin{pmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2k & -k & -k \\ -k & 2k & -k \\ -k & -k & 2k \end{pmatrix}$$